Strategic Formation of Credit Networks: Preliminary Report*

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Abstract

Recent models of abstract credit networks capture the dynamics of trust and obligations among agents over a series of transactions. Such networks support desirable characteristics in a distributed credit model, including robustness to intrusion, bounded risk, and the ability to support wide distributions of transaction pairings even with sparse direct credit relationships. We consider the question of how such networks may be formed in the first place, from decisions of autonomous self-interested agents to grant credit to others. We perform a theoretical analysis of a network contribution game in this framework under a simple scenario, where agents in the network transact only with other agents that they directly trust. Under this restriction we show that at Nash equilibrium, agents allocate credit budgets in a socially optimal manner. We also investigate via simulation another version of the network formation game, where agents may transact along paths in the credit network. In a model where agents have common beliefs about the default risk of others, we find that in equilibrium of the empirical game everyone issues credit to the same agents, yielding a network exhibiting a small set of central currencies.

1 Introduction

The study of strategic network formation seeks to understand the emergent behavior and properties of a network when agents establish connections to other agents in order to maximize their utility. Establishing a connection incurs a cost but also yields some value to agents connected through that edge. The agents are deemed to be utility-maximizing, that is, they make decisions in order to maximize the difference between their total value and their total cost. This problem has been studied in many different settings [Jackson and Wolinsky, 1996; Fabrikant et al., 2003; Corbo et al., 2006; Anshelevich and Hoefer, forthcoming]. One can ask interesting questions about the emergent properties of the networks formed in each setting: What topologies are feasible in equilibrium? What topologies maximize overall social welfare? How does this depend upon the cost of forming an edge and the value derived from having a connection? What is the price of anarchy? Can agents discover stable states quickly through some kind of iterated better-response dynamics?

This paper is a preliminary investigation into some of these questions in the context of credit networks. Credit networks represent a way of modeling trust and managing obligations among entities in a network. Nodes in the network print their own currency and maintain credit relationships representing the amount of each other’s currency they are committed to accept. These amounts can be viewed as the degree of trust one agent has for another. Credit commitments allow the network to serve as a decentralized payment infrastructure: arbitrary payments can be routed through the network by passing IOUs between trusting nodes in their respective currencies, thus obviating the need for a common currency. This model was introduced independently by DeFigueiredo and Barr [2005], Ghosh et al. (2007), and Karlan et al. [2009] as a generalization of budget constraints and as a mechanism for enabling transactions among untrusting agents in a network.

Prior work shows that a network with a reasonable amount of connectivity (liquidity) is sufficient to support and sustain transactions generated throughout the network [Dandekar et al., 2011]. In the current work we address how credit networks may be formed in the first place. We endogenize network formation by explicitly modeling the decision by each agent to issue credit to others. Issuing credit entails risk (a counterparty may violate the trust extended), as well as benefits (increases the probability that profitable transactions may be completed). A natural question is whether agents who rationally weigh these risks and benefits will actually form viable credit networks.

In this paper we provide some preliminary results on strategic formation of credit networks, through two approaches. First, we give theoretical results for a special case of the formation scenario, where agents’ trust in each other is captured via an underlying graph. An agent trusts its neighbors in the graph and is therefore willing to extend credit to others. The agents’ tolerance for exposure to risk in this model is captured via per-node credit budgets: the maximum credit that a node is willing to extend to others. An agent’s problem is to allocate its credit budget across its neighbors in order to maximize the long-term success probability of their transactions. We

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find that under certain technical conditions, including a restriction of transactions to occur directly between the parties receiving and issuing credit, the Nash equilibrium solutions correspond exactly to socially optimal credit allocations. Second, we investigate through empirical game-theoretic analysis an alternative model that allows transactions along paths of credit relationships. Here the agents’ problem is to choose how much credit to issue, and to whom, to balance the benefits of transaction profitability against the risk of credit default captured via per-node default probabilities. In this preliminary analysis we consider only a particular configuration of environment parameters and a set of eight heuristic strategies. We find that with a common prior on default probabilities (i.e., all nodes have identical estimates of the default probability of a given node), there is a strong tendency to form networks where everyone issues credit to the same small set of the most trustworthy nodes in the network. The networks that emerge in this setting are structurally similar to the centralized network described by Dandekar et al. [2011].

Both our theoretical and empirical results are preliminary, but they point to potentially important directions for a full investigation. Our theoretical analysis suggests the possibility that there may be a wider set of scenarios in which the price of anarchy of a credit network obtained endogenously by rational agents may be small compared to a centrally designed credit network. Our empirical analysis suggests an important direction for further modeling and investigation: if nodes do not have a shared prior on default probabilities, what kinds of credit networks emerge and how do they differ from a centralized currency system?

2 Background: A Model of Credit Networks

A credit network $G = (V, E)$ is a directed graph with $n$ nodes and $m$ edges. Nodes represent entities or agents. Edges represent pairwise credit limits between agents. An edge $(u, v) \in E$ has capacity $c_{uv} > 0$, which implies that $u$ has extended a credit line of $c_{uv}$ units to $v$ in $v$’s currency. If a node $y$ needs to pay $p$ units in its currency to node $z$ (for example, to buy a good that $z$ is selling), the payment can go through if the maximum credit flow from $z$ to $y$ is at least $p$ units. The payment will get routed through a chain of nodes from $y$ to $z$, where each link on the chain carries at least the requisite credit capacity. If an edge $(u, v)$ routes $p$ units of payment from $u$ to $v$, the credit capacity $c_{uv}$ increases by $p$ while $c_{vu}$ decreases by $p$. This update is illustrated by Figure 1. Thus routing payments in credit networks is identical to routing residual flows in general flow networks. Note that payment flows in the opposite direction of credit, so a payment merely results in a redistribution of credit; the total credit in the network remains unchanged.

This model is useful in settings where there is a need to model trust between nodes in a network. It is particularly well-suited for transactions in exchange economies such as P2P networks where it can be used to improve inefficiencies resulting from asynchronous demand and bilateral trading [Liu et al., 2010]. It has been used to thwart spam in e-mail and content-sharing systems such as YouTube [Mislove et al., 2008]. It can also be used in settings such as packet routing in mobile ad-hoc networks and combating spam in viral marketing over social networks. There is a large body of work in economics and sociology on social capital and favor exchanges in networks [Jackson et al., 2010]. This model provides a rigorous way of not only keeping track of favors owed to and by each individual in a network, but also a way to exchange favors, via trusted intermediaries, between individuals who do not know each other directly.

A natural question to ask is: How do nodes determine how much credit, if any, to extend each other under some notion of rationality? We assume a model of repeated transactions: nodes in a credit network probabilistically transact with each other at each time step. So the credit extended by a node $u$ to a node $v$ ultimately provides liquidity for node $u$ as well. That is, in addition to enabling $v$ to make payments by giving IOUs to $u$, it may also enable $u$ to make payments to $v$ since $v$ will take its own IOUs back. On the other hand, when $u$ extends, say, $p$ units of credit to $v$, $u$ exposes itself to the risk that $v$ might disappear leaving $u$ holding up to $p$ units of $v$’s irredeemable IOUs. Thus each node is faced with the strategic question of extending credit to maximize liquidity for itself while minimizing the risk it exposes itself to. We model risk in two different ways: in the theoretical analysis (Section 3) we model it as per-node budget constraints (each node has a limit on how much total credit it is willing to extend others), whereas in the simulations (Section 4) we model it as per-node default probabilities so that nodes try to minimize the expected loss from the default of their neighbors.

3 Theoretical Analysis

In this section we present a theoretical analysis of the credit network formation problem by formulating it as a network contribution game [Anshelevich and Hoefer, forthcoming]. We show that if we allow only symmetric bilateral transactions, that is, routing payments only along the direct edge between two transacting nodes, and interpret the long-term transaction success probability between adjacent nodes as their edge utilities, then for a broad class of stochastic transaction regimes the credit network formation problem is a network contribution game with concave, differentiable utilities. As a result, the price of anarchy in this game is 1. This means that, informally speaking, agents making selfish decisions based on local in-
3.1 Network Contribution Game

We frame the credit network formation problem as a network contribution game. Our setup is identical to that of Anshelevich and Hoefer [forthcoming], except we describe it in the language of credit networks. We take $G(V,E)$ as an exogenously defined simple undirected graph representing the underlying network. The $n$ nodes in $V$ represent agents, and the $m$ edges in $E$ represent relationships (e.g., friendships) between agents. Each node is constrained to extend credit only to nodes it trusts—its neighbors in $G$—in order to maximize its total utility. Extending credit to neighboring nodes transforms the given undirected graph $G$ into the directed graph with associated credit capacities described in Section 2. We model risk in terms of the maximum total credit that a node is willing to extend across its neighbors. Each node $u$ has an exogenously defined credit budget, $B_u > 0$, which represents the risk that node $u$ is willing to expose itself to. Nodes with higher budgets are more tolerant of risk, and vice versa.

More formally, node $u$’s strategy is a set of initial credit allocations $\{c_{uv}^0 \mid (u,v) \in E\}$ such that $c_{uv}^0 \geq 0$ and $\sum_{(u,v) \in E} c_{uv}^0 \leq B_u$. Let $\mathbb{R}^+$ be the set of nonnegative real numbers. Each relationship $e = (u,v)$ contributes utility $f_e : \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}_+$ to each node $u$ and $v$, where $f_e(\cdot,\cdot)$, is a non-decreasing function of $c_{uv}^0$ and $c_{vu}^0$. Note that the utility functions on any two edges may be different. Given the initial credit allocations of all nodes, the total utility of a node $u$, $w_u$, is simply the sum of utilities on each of its edges, that is, $w_u = \sum_{e=(u,v) \in E} f_e(c_{uv}^0,c_{vu}^0)$. A node $u$ distributes its credit budget $B_u$ among its neighbors in $G$ in order to maximize $w_u$. We measure the social welfare, $w_G$, of the network as the sum of utilities of all nodes in the network: $w_G = \sum_{u \in V} w_u$. It is easy to show that if the edge utilities, $f_e$, are concave and differentiable, this game has a price of anarchy of 1.

**Theorem 1.** If the edge utilities, $f_e(\cdot,\cdot), e \in E$, are concave and differentiable, the network contribution game has a price of anarchy equal to 1.

**Proof.** We know that network contribution games are potential games [Anshelevich and Hoefer, forthcoming]. In particular, the function $\Phi_G := w_G/2 = \sum_{e=(u,v) \in E} f_e(c_{uv}^0,c_{vu}^0)$ is a potential function. We also know that any Nash equilibrium of a potential game with a smooth concave potential is also a potential maximizer [Neyman, 1997]. If edge utilities, $f_e(\cdot,\cdot)$, are concave and differentiable, $\Phi_G$ is concave and differentiable. Therefore, any Nash equilibrium of the game must maximize $\Phi_G$. In other words, the price of anarchy of the game is 1. $\square$

The price of anarchy being 1 implies that Nash equilibria of the network contribution game are also optimal in terms of social welfare.

Note that we have not yet specified how nodes derive utility from extending credit to others. We would like to posit that nodes extend credit to each other in order to maximize the long-term success probability of their transactions. In order to do that, we first describe the model of repeated transactions under which we analyze the credit formation problem.

3.2 Transaction Model

We assume that each edge $e \in E$ is associated with a probability density function $g_e(\cdot)$ having support over $[0,\infty)$ and a corresponding cumulative distribution function $F_e(\cdot)$. Let $F_e(\cdot)$ be twice differentiable. The values of transactions along $e$ are drawn from the distribution given by $g_e(\cdot)$. Additionally, the transaction rate matrix $\Lambda = \{\lambda_{uv} : u, v \in V\}$ is given and satisfies the following properties:

- For all nodes $u \in V$, $\lambda_{uu} = 0$.
- For all nodes $u, v \in V$, $\lambda_{uv} > 0$ if and only if $(u,v) \in E$.
- For all nodes $u, v \in V$, $\lambda_{uv} = \lambda_{vu}$.
- And finally, $\sum_{u,v} \lambda_{uv} = 1$.

We allow only bilateral transactions, that is, routing payments only along the direct edge between two transacting nodes. At each time step $t$, we pick a node pair $(u,v)$ with probability $\lambda_{uv}$. We draw a transaction value, say $x^t$, from the distribution given by $g_e(\cdot)$ and try to route $x^t$ units of flow from $u$ to $v$ along edge $e$ (the flow is presumed to be a payment from $u$ to $v$). A transaction succeeds if $x^t \leq c_{uv}^t$ and fails otherwise. As described in Section 2, a successful transaction at $t$ results in $c_{uv}^{t+1} := c_{uv}^t + x^t$ and $c_{vu}^{t+1} := c_{vu}^t - x^t$. Note that for all $t > 0$, $c_{uv}^t + c_{vu}^t = c_{uv}^0 + c_{vu}^0 = c(e)$. For the rest of the section we denote by $\lambda_e$ the transaction rate between nodes $u$ and $v$ connected by edge $e$. These repeated probabilistic transactions along edge $e$ induce a Markov chain over $[0,\infty)$ which is governed entirely by $\lambda_e$ and $g_e(\cdot)$.

We will show that the steady-state transaction success probability, $q_{uv}$, between adjacent nodes $u$ and $v$ is an increasing, differentiable function of the total credit capacity, $c(e)$, along edge $e = (u,v)$. As a result, we can treat transaction success probability as a utility measure,

$$f_e(c_{uv}^0,c_{vu}^0) := q_{uv}(c_{uv}^0 + c_{vu}^0).$$

Further, we will show that that if $g_e(\cdot)$ is non-increasing, $q_{uv}(\cdot)$ is concave. Therefore, the credit formation problem in this setting can be cast as a network contribution game with concave, differentiable utilities.

3.3 Transaction Success Probability

We denote the Markov chain along edge $e$ by $M_e(\lambda_e, g_e) = \{X_e(t) \mid t \geq 0\}$. A state $X_e(t) = x$ of the Markov chain can be interpreted as $\lambda_{uv} = x$, $c_{uv} = c(e) - x$. Let $P_e$ be the transition kernel of the Markov chain and let $p_e$ be the corresponding density. Then $p_e$ is given by

$$\forall x, y, x, y \in [0, c(e)], p_e(x,y) = \begin{cases} \lambda_e g_e(x-y) & \text{if } x \neq y \\ (1-\lambda_e) + \lambda_e \int_x^\infty g_e(y)dy + \lambda_e \int_c^\infty g_e(y)dy & \text{if } x = y \end{cases}$$

Note that for all $0 \leq x, y \leq c(e), p_e(x,y) > 0$. It implies that $M_e$ is irreducible.
Lemma 2. Consider an edge \( e \in E \). The Markov chain \( \mathcal{M}_e(\lambda_e, g_e) \) is irreducible.

Proof. Observe that for any \( x \in [0, c(e)] \) and any closed interval \( A \subseteq [0, c(e)] \) of nonzero length, \( P^e_e(x, A) > 0 \). □

Lemma 3. Consider an edge \( e \in E \). The Markov chain \( \mathcal{M}_e(\lambda_e, g_e) \) has a uniform steady-state distribution over \([0, c(e)]\).

Proof. Consider a uniform distribution given by \( \pi_e \) such that \( \pi_e(x) = 1/c(e) \) for all \( x \in [0, c(e)] \). Then, \( \pi_e \) satisfies the detailed balance condition, \( \pi_e(x)p_e(x, y) = \pi_e(y)p_e(y, x) \) for all \( 0 \leq x, y \leq c(e) \). Therefore, \( \pi_e \) is a steady-state distribution of \( \mathcal{M}_e \). Further, since \( \mathcal{M}_e \) is irreducible, \( \pi_e \) is the unique steady-state distribution of \( \mathcal{M}_e \).

Let \( q_{uv}(c) \) be the steady-state success probability of the transaction where \( u \) pays \( v \) when the total credit capacity on edge \( (u, v) \) is \( c \).

Theorem 4. For adjacent nodes \( u, v \) such that \( e = (u, v) \),

\[
q_{uv}(c) = \frac{\lambda_e}{c} \int_0^c F_e(y)dy.
\]

Proof. Fix a time step \( t \). Assume that \( X_e(t-1) = y \), where \( 0 \leq y \leq c \). Then the transaction at time step \( t \) where \( u \) pays \( v \) would succeed if \( x^t \leq c - y \). So the success probability at time step \( t \) is given by \( \lambda_e\mathbb{P}(x^t \leq c - y) = \lambda_e F_e(c - y) \). Therefore, the steady-state success probability is given by

\[
q_{uv}(c) = \int_0^c \pi_e(y)\lambda_e F_e(c - y)dy
\]

\[
= \frac{\lambda_e}{c} \int_0^c F_e(c - y)dy,
\]

since \( \pi_e(y) = 1/c \) for all \( 0 \leq y \leq c \). Substituting \( x = c - y \), we get

\[
q_{uv}(c) = -\frac{\lambda_e}{c} \int_0^c F_e(x)dx = \frac{\lambda_e}{c} \int_0^c F_e(x)dx.
\]

Note that

\[
\int_0^c (c - y)g_e(y)dy = c \int_0^c g_e(y)dy - \int_0^c yg_e(y)dy
\]

\[
= cF_e(c) - \int_0^c yg_e(y)dy
\]

\[
= cF_e(c) - \left( yF_e(y)\right)_0^c - \int_0^c F_e(y)dy
\]

\[
= \int_0^c F_e(y)dy.
\]

Substituting \( \int_0^c (c - y)g_e(y)dy \) for \( \int_0^c F_e(y)dy \) in (2), we get

\[
\frac{dq_{uv}}{dc} = \frac{\lambda_e}{c} \left( -\frac{2}{c^3} \int_0^c yg_e(y)dy + \frac{1}{c} g_e(c) \right)
\]

\[
= \frac{\lambda_e}{c} \left( -\frac{2}{c^3} \int_0^c yg_e(y)dy + \frac{2}{c^3} g_e(c) \int_0^c ydy \right)
\]

\[
= \frac{2\lambda_e}{c^3} \int_0^c y(g_e(c) - g_e(y))dy.
\]

So, if \( g_e(y) \geq g_e(c) \) for all \( y \leq c \), then \( \frac{dq_{uv}}{dc} \) is non-positive and therefore \( q_{uv}(c) \) is concave. □

Note that many natural distributions have non-increasing density functions over \([0, \infty)\). Examples include the exponential distribution, the normal distribution \( \mathcal{N}(0, \sigma^2) \), and the power-law distribution.

4 Simulation-Based Game Analysis

In this section we present an experimental framework for evaluating another variant of the credit network formation problem. To explore the issue of trust in credit networks, we consider a scenario where there is explicit risk associated with issuing credit, namely that agents might default, exhausting their credit and leaving the network. Default in this scenario is not a strategic choice, but rather a probabilistic event like network payments. The strategic decision we focus on is that of the agents to issue initial credit. Our model is thus a one-shot game, in which every agent determines its credit allocation, upon which the stochastic sequence of payment and default events transpire according to specified probability distributions.
4.1 Environment
In our credit network game, a set of agents $V$ and a common knowledge event distribution $D$ are exogenously specified. $D$ gives probabilities for two types of interactions among players: credit events and default events. At each time step $t$, either a credit event or a default event occurs: $e(t) \in \{c, d\}$. A credit event $(b', s', x_{bs'}, s', x_{st})$ specifies a buyer and seller $b'$, $s' \in V$, as well as a cost $x_{st}$ to the seller of providing a good or service and a value $x_{bs'}$ to the buyer of receiving it. If paths through the credit network connecting $s$ to $b$ have total capacity $c' \geq x_{bs'}$, they transact, transferring credit $x_{st}$. A default event specifies a single agent $d' \in V$, who exhausts all available credit and leaves the network.

In the experiments described below, seller cost is constant, $x_{st'} = 1$, for all agents $s$ and all time steps $t$, and buyer value is drawn uniformly $x_{bs} \sim U[1.0, 1.2]$. At each time step, one event transpires: a default event with probability $\Pr(e(t) = d) = 0.002$, or a credit event with probability $\Pr(e(t) = c) = 0.998$. In a default event, one agent $d'$ is selected according to a known probability. This probability comes from assigning each agent a weight drawn uniformly from $[0, 1]$ and normalizing these weights to sum to 1. These probabilities are not updated as agents leave the network, so if an agent that has already defaulted gets selected, no agent defaults in that time step. In a credit event, a pair of agents $(b', s')$ is selected according to a known probability distribution. This probability comes from assigning each pair a weight drawn from a Pareto distribution with shape parameter $\alpha = 2$.

An action for agent $v \in V$ consists of an initial credit allocation $c_{vua}$ for every other agent $u \in V$. Once credit allocations are determined, agent interactions are simulated for 10,000 time steps according to the event distribution. An agent’s payoff after a series of events is the total value of services received minus services provided across all credit events, less any losses from default events,

$$\text{payoff}(v) = \sum_{t | b' = v} x_{bs'} - \sum_{t | s' = v} x_{st} - \sum_{t | d' = u} c'_{vua}.$$  

Note that agents are not credited with the amount of the links they exhaust upon defaulting. This payoff varies according to the particular draws from the event distribution, so in our study we simulate the model many times and estimate payoffs by the sample average.

4.2 Empirical Game-Theoretic Analysis Method
Because this model is intractable to solve analytically, we undertake an empirical game-theoretic analysis (EGTA) [Wellman, 2006]. In the EGTA approach, we use simulation to estimate a game form over a set of heuristic strategies. A strategy for the credit network formation game can be any function from the event distribution to an initial credit allocation. To render simulation-based analysis tractable, we enumerate a manageable number of potentially interesting heuristic strategies, as described in Section 4.3.

For an $N$-player symmetric game over $S$ strategies, there are $N(S-1)$ distinct profiles. Our simulated credit network has 60 nodes, each corresponding to an agent. Since a 60-player game presents a computationally infeasible profile space for even a few strategies, we abstract the model to a simpler five-player game using hierarchical reduction [Wellman et al., 2005]. In this reduction, each player of the game chooses a strategy to be implemented by a 12-agent coalition in the original network. Because the players choose heuristic strategies before their types are known, the game is in fact symmetric, and we can find a symmetric mixed-strategy Nash equilibrium using replicator dynamics [Cheng et al., 2004].

To estimate the empirical game, we computed the payoffs for a given profile by drawing an assignment of default weights for each agent and transaction weights for each pair. We then apply the agents’ strategies to determine initial credit assignments, and simulate credit and default events according to $D$ for 10,000 time steps on the resulting credit network. The payoff for each profile is estimated by the average over 30 such simulation runs.

4.3 Strategies
We conducted three empirical game experiments. The first included the following five strategies:

- **Zero Credit** The Zero strategy extends no credit, but can still participate in the credit network as long as other agents extend credit to it. Agents following this strategy will never have a negative payoff, but could miss out on many potential transactions.

- **Random Links** The Random strategy extends $c = 5$ units of credit to each other agent with probability $\frac{1}{4}$. This strategy extends significantly more credit links and issues significantly more total credit than any of the others we tested.

- **Low Default Risk** The Low Default strategy extends 5 units of credit to each other agent whose probability of defaulting is below a threshold (specifically, agents $v$ such that $\Pr(d' = u \mid e(t) = d) < 0.004$, meaning that over the course of the 10,000 event simulation run $v$ has less than an 8% chance of ever defaulting). This strategy takes the most conservative step away from Zero, allowing some liquidity with minimal risk.

- **High Transaction Probability** The High Transact strategy extends 5 units of credit to each other agent with whom the creditor has a high probability of transacting as a buyer ($\Pr(b' = u, s' = v \mid e(t) = c) > 0.0005$, meaning that at least 5 transactions where $u$ buys from $v$ are expected over the 10,000 event simulation run). This strategy seeks to create liquidity to enable the trades the agent considers most probable.

- **High Expected Value** The High EV strategy extends 5 units of credit to the agents from whom the creditor expects to gain the most value. This value for agent $u$ of issuing credit $c$ to agent $v$ is calculated as the expected value of transactions where $u$ buys from $v$, minus the expected cost of transactions where $u$ sells to $v$, minus an estimate of the loss from the possibility that $v$ defaults:

$$E[U_{uv}] = \sum_{b' = u, s' = v} x_{bs'} - \sum_{s' = u, b' = v} x_{st} - \sum_{v \text{ defaults}} c'_{vua} \cdot \Pr(v \text{ defaults}),$$
where $c_{0}^{u,v} = 5$ is the initial amount of credit that would be issued. The last term does not perfectly characterize the cost of defaults, because the actual amount of outstanding credit at the time of default can vary based on any transaction in the network which uses this link. In the first experiment, the threshold for issuing credit is set at $E[U_{u,v}] > 0.5$.

The second experiment added the following two strategies, for a total of seven:

- **High Transact and Low Default** The $HT \land LD$ strategy starts by eliminating all agents with very high probability of defaulting (the threshold is set much higher than for $Low\_Default$, at $Pr(v \text{ defaults}) < 0.33$ over 10,000 rounds), and then issues 5 units of credit to those among the remaining agents those with whom the creditor is most likely to transact ($Pr(b^t = u, s^t = v | e(t) = c) > 0.0004$, or at least four transactions expected).

- **High EV and Low Default** The $EV \lor LD$ strategy eliminates the same high-default-risk agents from consideration, but then uses the method as High$_{EV}$ to select which of the remaining agents to issue 5 units of credit. This strategy sets its threshold lower than High$_{EV}$, at $E[U_{u,v}] > 0$.

The third experiment involved just four strategies: Zero, Random, Low$_{Default}$, and $EV \lor LD$:

- **High EV or Low Default** The $EV \lor LD$ strategy offers 5 units of credit to agents with especially low default probability ($Pr(d^t = u | e(t) = d) < 0.002$, or at most 4% cumulative default probability over 10,000 rounds) and also to agents with especially high expected net transaction value ($E[U_{u,v}] > 4$).

Note that all agents that issue credit do so in amounts of exactly 5 units, and that the various threshold parameters of those strategies (excluding Random) were calibrated so that on average each agent would issue credit to 5–7 others. We intend to loosen these restrictions in future experiments.

### 4.4 Results

In the first five-strategy experiment, replicator dynamics converged (from any initial mixture) to a pure strategy Nash equilibrium in which all players chose Low$_{Default}$. When all players adopt this strategy, all agents extend credit to the same small group of highly trustworthy agents (usually 5–7 of them). The resulting network, pictured in Figure 2, bears some resemblance to the centralized currency credit network described by Dandekar et al. [2011], in that agents give a large amount of credit to the central subgraph, and transactions can occur only using the currency of one of those central agents.

Following this result, we ran a second experiment which added two strategies that tried to improve on Low$_{Default}$. These strategies use default probability as a filter to eliminate high-risk agents, and then apply criteria of transaction probability or expected value to make a final selection of how to allocate credit. However, we once again found that replicator dynamics converged to an equilibrium of everyone playing Low$_{Default}$. This suggests that the value of playing Low$_{Default}$ stems not only from avoiding the cost of defaults, but also from how it enables agents to coordinate in connecting to the same subgraph.

In the third experiment, we attempted again to find a strategy that would improve on Low$_{Default}$. The idea behind the $EV \lor LD$ strategy was that it might allow much the same networks stability as Low$_{Default}$, while potentially increasing liquidity on highly profitable paths. But again we find that only Low$_{Default}$ is played with positive probability in equilibrium.

For all three of these empirical games we measured NE regret [Jordan, 2010], the amount lost by one player when deviating to a given strategy from an $\epsilon$-Nash equilibrium, in this case, the equilibrium in which all players choose Low$_{Default}$. We also measured the payoff and the regret of each pure strategy profile consisting entirely of one strategy. The regret of a pure strategy profile is the maximum gain a player could achieve by deviating from that profile to any other strategy. The results are presented in Table 1, with strategies ranked by NE regret.

<table>
<thead>
<tr>
<th>heuristic strategy</th>
<th>NE regret</th>
<th>s.p. regret</th>
<th>s.p. payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low$_{Default}$</td>
<td>0.0</td>
<td>0.0</td>
<td>6.8</td>
</tr>
<tr>
<td>HT$\land LD$</td>
<td>0.2</td>
<td>0.9</td>
<td>6.2</td>
</tr>
<tr>
<td>EV$\land LD$</td>
<td>0.8</td>
<td>1.0</td>
<td>6.6</td>
</tr>
<tr>
<td>EV$\lor LD$</td>
<td>0.9</td>
<td>0.9</td>
<td>6.8</td>
</tr>
<tr>
<td>High$_{Transact}$</td>
<td>2.0</td>
<td>3.5</td>
<td>4.1</td>
</tr>
<tr>
<td>High$_{EV}$</td>
<td>3.1</td>
<td>4.7</td>
<td>5.1</td>
</tr>
<tr>
<td>Zero</td>
<td>5.9</td>
<td>1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Random</td>
<td>10.9</td>
<td>14.1</td>
<td>-5.0</td>
</tr>
</tbody>
</table>

Table 1: Average NE regret, symmetric profile regret, and symmetric profile payoff of each strategy.

### 4.5 Discussion

In all of our EGTA experiments, players consistently chose the Low$_{Default}$ strategy in equilibrium. This strategy results in a centralized credit network structure in which a few highly-connected agents facilitate trade among the rest, similar in spirit to the central currency model described by Dandekar et al. [2011]. These central agents are attractive targets for credit
allocations for two reasons. First, they have relatively low default risk. Second, given that everyone else is issuing them credit, a link from any one of them creates a path to every other node in the network. Consider a source (yellow) node in Figure 2. Initially it has no credit, and cannot initiate any transactions as a buyer. As soon as any blue node buys from it, however, it obtains an incoming link, which opens up directed paths to purchase from any other node. In other words, the fact that blue nodes are universally trusted makes them central to facilitating trade in the network.

The ability to coordinate on centralized currencies using the Low_Default strategy depends on our assumption that agents have a common prior on the default events (i.e., all nodes assign the same default probability to a given node). It remains open for future work to establish whether strategies that coordinate on central currencies will emerge in situations where agents have heterogeneous beliefs about default, or where defaults themselves are conditional on the identity of neighbors.

It is also clear that even in the current setup, there could be other equilibria under some possible settings of our environment parameters. If the cost or probability of defaults were substantially higher, agents would hardly ever have incentive to extend credit and the Zero strategy would dominate. If the surplus from transactions were substantially larger, their value might outweigh the various costs to the point where agents might wish to extend credit to everyone to maximize liquidity. Similar effects might also result from varying the length of the simulation.

We plan to experimentally study the setting where, as in the model of Section 3, a network is exogenously given. Since edges in the network represent pre-existing relationships, adjacent nodes can more precisely evaluate each other’s default probabilities than can non-neighbors. Thus nodes will be more inclined to extend credit to their neighbors in the graph. This model could also be generalized so that nodes may assign arbitrary default probabilities to each other in a pairwise manner.

5 Conclusion

We report progress on our investigation of strategic issues in the formation of credit networks. Our work to date has proceeded on two tracks, one based on theoretical examination of an analytic model, and the other emphasizing empirical analysis of simulation experiments. Both employ game-theoretic principles, and aim at characterizing the network of credit relationships that can arise from rational and autonomous credit-granting decisions.

The theoretical analysis treats a network contribution game, where granting of credit is a form of contributing to network liquidity. Our model restricts attention to direct bilateral credit transactions, but in return achieves general welfare results with qualitative conditions on the value distributions over transactions. In particular, we identify a class of environments where Nash equilibria and social welfare maximization coincide, which means that self-interested agents can achieve globally desirable outcomes.

The simulation-based analysis covers a more complex transaction environment, where payments may be routed along paths of credit relationships. The question on this track is about how networks get formed when agents’ credit-granting decisions trade off the goals of facilitating profitable transactions and avoiding default risk. We employ empirical game-theoretic methods to identify strategically stable configurations over a set of heuristic strategies. Results from our simulations so far support the notion that if there is a shared prior on default probabilities of nodes, a centralized network emerges where a few trustworthy nodes facilitate trade in the network. It leaves open the question of what kind of network is formed when nodes do not have a common prior on default probabilities.

There is a great deal of further work required to establish our results in more general settings, and answer further strategic questions about formation of credit networks. We plan to continue our investigations on both tracks presented here, and seek ways to shape each track by the results of the other as we proceed.

References


