A Strategic Analysis of Portfolio Compression

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ABSTRACT

Portfolio compression, the elimination of debt cycles in a financial network, is employed in over-the-counter derivative markets as a method for simplifying balance sheets. Canceling debts through compression can sometimes promote financial stability by stopping the spread of contagion from an insolvent firm hit by a negative financial shock. However, previous work has demonstrated that in some cases compression can exacerbate systemic risk by removing paths of shock absorption. We analyze portfolio compression as a strategic decision made by firms within a debt network. We define a network game in which firms represented as nodes have only local information and we ask what criteria the firms should consider in their decision to compress. We propose a variety of heuristic strategies and evaluate them using agent-based simulation and empirical game-theoretic analysis. In our experiments, compression may or may not benefit nodes on a cycle, and our results show that simple strategies based on local information can effectively improve firms’ decision making. We further examine which features are most useful under various conditions, and find that the results depend on the rate at which assets can be recovered from insolvent nodes. When recovery rates are low, firms focus on avoiding contagion, and when they are high they consider the benefits of retaining cycles to cushion shocks. Finally, we then analyze the effects on systemic risk in equilibrium, finding that when nodes strategically choose to compress, the outcome is more likely to be beneficial than harmful.

KEYWORDS

financial network, portfolio compression, game theory

ACM Reference Format:

1 INTRODUCTION

The idea of portfolio compression is to simplify a web of financial obligations by canceling a cycle of debt, leaving each party’s net position the same. Consider the situation in Figure 1 where financial institution A owes money to institution B, B owes a like amount to institution C, and C owes the same amount to A. Furthermore, suppose the debt contracts comprising the cycle over these institutions have the same interest rates and maturity dates. Intuitively, canceling the three debts leaves all parties in the same net position, while simplifying their balance sheets.

If all financial institutions on a cycle are solvent and able to pay their debts when due, the operation of compressing portfolios by canceling the debts has no effect and the compression decision is of little interest. With risk of default, however, portfolio compression is not generally neutral and may affect stability in the network.

For instance, if institution A in Figure 1 turns out to be insolvent, then B would be better off had the compression occurred than had it not. C might similarly be better off if B’s ability to pay its debt depends on receiving payment from A. Thus, compression may serve to limit contagion in financial networks by removing paths of default propagation. This potential to limit financial contagion while simplifying balance sheets is the reason compression has gained prominence since the 2008 financial crisis.

On the other hand, portfolio compression can also impede the ability for parts of the network to absorb losses from an insolvent institution, thus enabling spread that may have been avoided. Schuldenzucker and Seuken [2020] present an example, shown in Figure 2, where eliminating a cycle causes previously solvent institutions to become insolvent. The network starts with a cycle like the one described above with the addition of two edges: A owes money to D and D owes money to E. Each institution holds some quantity of external assets outside of the network. Suppose the cycle were compressed and then A experiences a shock that reduces its external assets to zero. In this scenario, A would have
no assets to pay its remaining creditor, D. To absorb the loss from A’s insolvency, D must hold enough external assets to pay E, otherwise D would also default. In contrast, suppose this cycle were not compressed prior to the shock reducing A’s external assets to zero. If B’s external asset holdings are sufficiently large, B could absorb the loss from A’s insolvency and prevent its spread to C. In turn, C would pay A the full debt amount. A would then use its internal assets to make a partial payment to its creditors, which could prevent D from becoming insolvent. Note that the realization of this outcome depends on the value of the partial payment made by A. In this case, the cycle cushions the shock and so compression has the harmful effect of allowing default to spread elsewhere.

Although compression in this case is harmful to the network, the optimal outcome of rejecting the compression may not be realized. In this example, institution B would prefer compression, while A and C’s indifference towards compression may result in their acceptance of the proposal. However, imagine a slight modification to the network so there is a liability from D to C. If D’s ability to pay its liabilities depends on receiving payment from A, compression would allow insolventy to spread to C. In this case, C would no longer be indifferent about the compression. Thus, we can see that the qualitative effects of compression are dependent on characteristics of the network.

Some of the prior work on compression has focused on analyzing which characteristics of a network determine whether a compression will be harmful or not. One important factor is the recovery rate: the fraction of assets an insolvent node is able to recover and use to pay back creditors, with the remaining assets being lost to default. Verraart [2019] finds that when recovery rates are nonzero, portfolio compression may be harmful to the network. Schuldenzucker and Seuken [2020] build on these results by studying a broader range of network aspects to determine when financial institutions are incentivized to perform compression and when compression is a Pareto improvement.

These analyses illuminate the interplay of network structure and effects of compression, from an ex-post perspective where details of the external shock are known. In practice, decisions about whether to compress debts are made by institutions with a limited view of the network and prior to resolution of uncertain financial events. Evaluation of compression decisions, therefore, necessitates a model accounting for uncertainty and incomplete information on the state of the network. Further, prediction of what these firms would decide should also consider strategic effects, recognizing that optimal policies generally depend on how the other firms make their decisions.

To analyze the broader context of compression decisions, we define a one-shot game on a financial network, with nodes representing financial institutions connected by directed edges representing debt obligations. The strategic decision facing nodes is whether to accept or decline a proposed compression. We consider a variety of heuristic strategies based on the information available to nodes about the network, which may provide useful indications of potential negative effects of compression to the voting node. To evaluate these strategies experimentally, we generate a large number of random financial networks with debt cycles, and employ agent-based simulation to determine compression decisions and outcomes at different insolvent node recovery rates. Following the methodology of empirical game-theoretic analysis (EGTA) [Tuyls et al. 2020; Wellman 2016], we identify Nash equilibria among the heuristic strategies.

We offer three main contributions in this work:

1. framing portfolio compression as a strategic decision by individual agents with incomplete information;
2. an agent-based simulation framework to support strategic analysis; and
3. analysis of the effects on systemic risk of strategic compression decision making, for a particular debt network scenario.

Within this environment, we have found evidence that for an individual node, the optimal policy is generally conditional on features of the local network and the individual agent’s balance sheet. Through a novel use of EGTA methodology, we find that at higher recovery rates, there is a greater potential for cycles to cushion shocks in the network. When the compression decision is made strategically, the likelihood of a given cycle being compressed is lower than in the social optimum. Compression decisions generated by strategic consensus may be better or worse than no compression. However, the price of anarchy of a strategic compression decision is high for all recovery rates.

2 RELATED WORK

Eisenberg and Noe [2001] pioneered the modeling of financial debt networks as a set of nodes representing banks, with directed edges forming interbank liabilities, for the purpose of studying systemic risk. Rogers and Veraart [2013] expand the representation by adding recovery rates applied to insolvent nodes. They use their expanded model of the financial debt network and clearing algorithm to study the rescue of failing banks in the network through mergers with solvent banks. The authors find that solvent banks do not always have both the means and incentives to rescue insolvent banks in the network.

Prior studies of contagion and systemic risk in financial networks have emphasized network structure [Acemoglu et al. 2015; Elliott et al. 2014; Glasserman and Young 2015]. These works do not specifically address the issue of debt compression. Prior studies of compression have tended to focus on algorithms for identifying candidate compressions, rather than the decision of whether to compress particular candidates [D’Errico and Roukny 2019; O’Kane 2017]. For example, D’Errico and Roukny [2019] use transaction data on over-the-counter derivatives called credit default swaps (CDSs) to study efficiency of different types of portfolio compression. A CDS is a contract that enables a creditor to hedge the risk of a debt obligation with another member of the network. Efficiency of a compression method is determined by the amount of the excess—the total amount eligible for compression—a compression method is able to eliminate. Of the types of compression analyzed, conservative and hybrid compression are the most efficient at removing excess from the network.
Veraart [2019] analyzes the effect of portfolio compression on systemic risk, specifically focusing on the relationship between systemic risk and recovery rate. Systemic risk is defined by the set of banks that become insolvent with compression compared to the set in the same network without any compression. Compression is defined as beneficial when the set of banks defaulting with compression is a strict subset of the banks defaulting without compression, weakly beneficial when beneficial or the two sets are the same, and harmful if some node defaults under compression which would not have defaulted without compression. Veraart proves that when the recovery rate is nonzero, compression can be harmful to the network. Conversely, compression with a zero recovery rate is always weakly beneficial. Furthermore, when no nodes on the cycle become insolvent, the network is indifferent to compression.

Schuldenzucker and Seukon [2020] study compression from the ex-post perspective, analyzing the relationship between a variety of network characteristics and the effects of compression on systemic risk. The analysis focuses on whether compression is beneficial, either in a Pareto or social welfare sense. For example, they establish a connection between homogeneity of the network and compression being a Pareto improvement.

These analyses of portfolio compression have demonstrated the potential benefits of compression both quantitatively and qualitatively, as well as potential harm to systemic risk. Given that compression has the potential to have a positive or negative effect on the financial network, it seems worthwhile to consider how compression has the potential to have a positive or negative effect on systemic risk. The analysis focuses on whether compression is beneficial, either in a Pareto or social welfare sense. For example, they establish a connection between homogeneity of the network and compression being a Pareto improvement.

3 GAME MODEL

Adopting the ex-ante perspective, we introduce a compression game where agents in a financial network decide whether to adopt a given compression proposal using heuristic strategies based on local information. We start by describing the financial network environment and the strategy set before describing the game in full.

3.1 Financial Network

The financial network model comprises \( N = \{1, \ldots, n\} \) nodes each representing a financial institution and a set of directed edges, \( E \), representing debts between institutions. Edge \((i, j, a) \in E\) represents a liability of value \( a \) owed by \( N_i \) to \( N_j \). The same edge represents an asset for node \( N_j \), and since it is within the network we term it an internal asset.

To allow for multiple debt edges between a pair of nodes \( N_i \) and \( N_j \), we denote the overall liability between nodes by \( l_{ij} = \sum_{(i, j, a) \in E} a \). Thus \( N_i \)'s total liabilities \( L_i \) are given by:

\[
L_i = \sum_{j=1}^{n} l_{ij}.
\]

Each node is also endowed with some external assets \( e_i \) and we define \( N_i \)'s total assets \( A_i \) as the sum of external and internal assets:

\[
A_i = e_i + \sum_{j=1}^{n} l_{ji}.
\]

A node is considered insolvent when \( A_i - L_i < 0 \).

Using the financial network definitions, we can formally define a portfolio compression. Given a cycle \( c = (N^c, E^c) \) with \( N^c \subseteq N \) and \( E^c \subseteq E \), let \( \mu^c = \min_{(i, j, a) \in E^c} a \) denote the smallest liability on the cycle. Then a compression of cycle \( c \) is defined by the revised set of liabilities for edges on the cycle:

\[
\forall (i, j, a) \in E^c, \quad a^c = a - \mu^c.
\]

It can be seen that compression reduces the assets and liabilities of the nodes on the cycle while maintaining their net positions.

![Figure 3: A simple debt cycle before and after compression by two units.](Image)

An example of a potential cycle and its subsequent compression can be seen in Figure 3. The cycle is \( c = ([A, B, C], [(A, B, 5), (B, C, 3), (C, A, 2)]) \) with \( \mu^c = 2 \). Compressing the cycle allows node A to eliminate its exposure to node C and decreases the liabilities of A and B.

A financial network is also associated with recovery rate parameters \( \alpha, \beta \in [0, 1] \), which govern payments of an insolvent node when its debts reach maturity. The \( \alpha \) parameter denotes the fraction of an insolvent node’s external assets that are recovered. \( \beta \) controls what fraction of an insolvent node’s internal assets are recovered. The assets not recovered are lost to default costs. The total assets recovered by an insolvent node are:

\[
\alpha e_i + \beta \sum_{j=1}^{n} l_{ji}.
\]

We make several simplifying assumptions for the current analysis. The first is that all debts in the network have the same maturity date and interest rate. We also assume a financial institution never has a liability to itself, \( l_{ii} = 0 \). Finally, we assume that \( \alpha = \beta \), and refer to \( \alpha \) as the parameter controlling the fraction of an insolvent node’s total assets that are recovered in the event of a default.

3.2 Strategies

The decision of interest for our analysis is a node’s choice to accept or reject a proposal to compress a given debt cycle. In principle, nodes may take into account anything they know about the network. For our study, we define a set of heuristic strategies that consider different forms of information about a node’s situation and the financial network, including various measures of assets and liabilities, overall and associated with particular nodes and edges. We then analyze the strategic interaction of nodes on a financial network in their selection from among the available heuristics.

The strategies provided are specified in Table 1. The table lists positive versions of the strategies, defining the condition under which node \( N_i \) chooses to accept the proposed compression. We

\[1\text{Under this assumption we can also limit each ordered pair of nodes to at most one debt edge.}\]
also include the corresponding negations, where a node rejects the compression under the same condition.

Table 1: Heuristic strategies for accepting compression proposals. The corresponding negations (Vote "no" iff) are also available.

<table>
<thead>
<tr>
<th>Strategy Name</th>
<th>Vote &quot;yes&quot; iff</th>
</tr>
</thead>
<tbody>
<tr>
<td>YES</td>
<td>Always</td>
</tr>
<tr>
<td>Asset Ratio</td>
<td>$\sum_{i=1}^{n} \frac{l_{ij}}{A_i} &gt; 0.5$</td>
</tr>
<tr>
<td>Expected Internal</td>
<td>$0.5 \times (\sum_{j=1}^{n} l_{ij}) - L_i &lt; 0$</td>
</tr>
<tr>
<td>External</td>
<td>$e_j - L_i &lt; 0$</td>
</tr>
<tr>
<td>Internal Ratio</td>
<td>$\frac{l_{ij} - \mu^e}{\sum_{j=1}^{n} l_{ij}} &gt; 0.5$</td>
</tr>
<tr>
<td>Internal</td>
<td>$(\sum_{j=1}^{n} l_{ij}) - L_i &lt; 0$</td>
</tr>
<tr>
<td>Liability Ratio</td>
<td>$l_{ij} - \mu^e &gt; 0.5$</td>
</tr>
<tr>
<td>Relative Liability</td>
<td>$\frac{l_{ij} - \mu^e}{A_j} &gt; \frac{l_{ij} - \mu^e}{L_i}$</td>
</tr>
</tbody>
</table>

The first strategy listed in Table 1 is unconditional acceptance, a strategy we label YES. This is meant to cover as a baseline the common practice of simply accepting any compression proposed. The tendency to agree may be due to a number of factors, including a desire to simplify balance sheets or to comply with regulation [Schuldenzucker and Seuken 2020]. For symmetry we also include its complement, the unconditional reject strategy NO. The remaining heuristic strategies are defined by inequalities over assets and liabilities in various ways. The complements are defined by simple negation, or flipping the inequality. For example, the Asset Ratio strategy, $\sum_{i=1}^{n} \frac{l_{ij}}{A_i} > 0.5$, has a negation Neg Asset Ratio, $\sum_{i=1}^{n} \frac{l_{ij}}{A_i} \leq 0.5$.

Our collection of strategies is far from exhaustive, but is designed to cover a variety of strategically relevant features. Understanding the relative usefulness of different network features is the main object of our game-theoretic analysis.

3.3 Network Game

We define an $n$-player compression decision game, in which the agents are each assigned to one node of a randomly generated financial network. Agents select a heuristic strategy from the set available. A compression candidate is identified, and the agents on the cycle decide whether to accept or reject the compression using their selected strategy. If all accept, the cycle is compressed. After a negative external asset shock, agents receive a payoff based on the resolution of liabilities in the network.

More specifically, we generate $n$ nodes and assign agents to them uniformly at random. Each node $N_i$ is given a random endowment of external assets, $e_i \sim U(10, \ldots, 20)$. We then create liabilities within the network by the following procedure:

- Randomly select two nodes $N_i$ and $N_j$.
- If $l_{ij} = 0$ and $e_j > 0$, add edge $(i, j, v)$ to $E$, with $v \sim U(1, \ldots, e_j)$.
- Update external assets: $e_j \leftarrow e_j + v$, $e_j \leftarrow e_j - v$.

The procedure repeats until the network contains at least one cycle of three or more edges. Note that throughout, all nodes in the network are solvent by construction.

The game proceeds with a compression vote. For cycle $c$, the agents assigned to the $N^c$ nodes independently cast their vote for the proposed compression cycle using their selected strategy. A compression is performed if and only if all $N^c$ nodes vote in favor of the compression. If a network contains more than one cycle, they are addressed in sequence. We consider compression on cycles with $|N^c| \geq 3$ before cycles with $|N^c| \geq 2$. Cycles within these size classes are ordered randomly. The order is relevant, as decisions for a given cycle may be affected by the outcome of compressing cycles considered earlier in the sequence.

As our goal is to understand the effect of strategic compression on systemic risk, we follow the compression decisions with a financial stress event. Absent an insolvent node, compression has no effect on systemic risk [Veraart 2019, Theorem 3.10]. We therefore subject the network to a shock on external assets, designed to throw some nodes into insolvency. Our asset shock is modeled as the realized realization of outside investments. Nodes invest their external assets that remain after liability creation in one of two ways. Nodes not on a cycle are randomly assigned to either one of the two investments or to not make an investment, each with probability $1/3$. Nodes on a cycle are randomly assigned to one of two investments, each with probability $1/2$. This increases the likelihood that a non-empty subset of nodes on a cycle become insolvent, aligning with a result of which states that compression does not affect systemic risk if no nodes on a cycle become insolvent. In the final step, the external shock is triggered by uniformly, randomly choosing an investment option to fail. When an investment fails, nodes assigned to that investment do not recoup any of their investment.

We then resolve the network by making all nodes pay off their debts. Solvent nodes pay in full and insolvent nodes pay creditors from their recovered assets proportional to their respective liabilities. Following the algorithm by Rogers and Veraart [2013], we define a proportional liability matrix $\Pi$ with entries $\Pi_{ij} = \frac{l_{ij}}{L_i}$. The payment $p$ made by an insolvent bank $N_i$ to one of its creditors $N_j$ is then given by:

$$p_{ij} = a \left( e_i + \sum_{k=1}^{n} p_{ki} \right) \Pi_{ij}.$$  

At the end of the game, the payoff to the agent assigned to node $N_i$ for playing the selected strategy is its remaining asset holdings after resolving all liabilities. Agents for nodes that become insolvent receive zero payoff.

4 EMPIRICAL GAME-THEORETIC ANALYSIS

We analyze the compression game by extensive simulation of selected strategy combinations through a process known as EGTA. A strategy profile lists strategies and how many of the $n$ players employ each strategy. Strategy profiles are evaluated by repeated simulation of the strategy combination on instances of the compression game. Each run comprises the steps above: generating a
network (including random assignment of players to nodes), identifying compression candidates, then for each candidate conducting a compression vote followed by compression if all nodes on the cycle accept the proposal, and finally resolving the outcome after an external shock. The estimated payoff vector for the profile is the sample average of payoffs observed over many simulation runs.

We follow an iterative procedure for selecting profiles to simulate, geared toward identifying symmetric mixed-strategy Nash equilibria, using methods similar to those employed in prior EGTA studies [Cassell and Wellman 2013; Wellman et al. 2013]. The games in this study have $n = 10$ players. With 16 strategies there are over 3.2 million distinct profiles in this game, so we can practically evaluate only a fraction of these. Fortunately, we are generally able to identify and confirm approximate equilibria far short of an exhaustive evaluation of the profile space [Fearnley et al. 2015].

Through this approach, we estimate an empirical game over a manageable profile space for six games, defined by varying settings of the recovery rate: $\alpha \in \{0, 0.1, 0.3, 0.5, 0.7, 1.0\}$. For each we identify one or more symmetric Nash equilibria: mixed profiles for which none of the pure strategies is a beneficial deviation. Table 2 presents the extent of simulation employed for the EGTA computation across the six games.

Our goal was to identify the local information that is most useful for making the compression decision. To this end, we group strategies that share similar characteristics into sets of features. The first feature is Assets and includes the strategies asset ratio, expected internal, external, internal ratio, internal, and their negations. The second feature, Liabilities, encompasses the liability ratio and relative liability strategies, as well as their negations. The third feature, Dynamic, includes strategies that consider changes that may occur in the network in their decision making. These include: internal ratio, liability ratio, relative liability, expected internal, and their negations. Finally, we define the feature Unconditional, comprising the strategies YES and NO.

To evaluate the usefulness of different features for making the compression decision, we solve the empirical games for each $\alpha$ value and analyze the effect of including or excluding strategies that use these features.

### 4.1 Equilibria

For a given mixed profile, the weight of a strategy is the probability it is played. The weight of a feature is the sum of weights for strategies using that feature. By these measures, we can quantify the weight of features in equilibrium profiles of each game. For games with multiple equilibria found, we report the average feature weights across the equilibrium profiles.

The results of this analysis are shown in Figure 4. As the features may comprise overlapping sets of strategies, weights may exceed one for some games. Generally speaking, we find that strategies employing simple, local network information are preferred to the unconditional strategies by nodes making the compression decision. The clear exception occurs at $\alpha = 0$, for which YES is a pure Nash equilibrium, which aligns with the theorem stating compression cannot be harmful for zero $\alpha$ [Veraart 2019]. Strategies using Assets or Dynamic features are assigned the majority of weight in equilibrium for games with $\alpha > 0$.

### 4.2 Feature Analysis

The second analysis using the features gauges the usefulness of each feature by the maximum payoff gain a player receives for deviating to a strategy in the feature set. This allows us to measure the value of access to certain pieces of information about the network. For this analysis, we first identify an equilibrium of a constrained version of the game that excludes strategies in the feature being tested. We then measure the gains associated with one-player deviations from this equilibrium to a strategy in the feature set excluded. The maximum payoff gain is defined as the greatest increase over the constrained equilibrium payoff achievable by deviating to an excluded strategy.

Figure 5 displays the results from this analysis. From these results, we can see that gaining the ability to use the Assets feature, strategies that use asset holdings, or the Dynamic feature, strategies that consider how a negative shock or compression might affect the network, offer the largest gains.

This analysis focuses on the case where all other information, except the feature is known. However, we can also examine the case where an agent has no information and see which feature might be most important in making the compression decision. This analysis is run similar to the above description, except the initial strategy set contains only the unconditional strategies. The one-player deviations being tested contain all the other features.

### Table 2: Simulation effort for EGTA applied to six games. For each game, we tabulate the minimum number of runs per profile (M=million) and the number of profiles evaluated.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>min # runs/profile</th>
<th># profiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.5M</td>
<td>12,109</td>
</tr>
<tr>
<td>0.1</td>
<td>1.0M</td>
<td>78,247</td>
</tr>
<tr>
<td>0.3</td>
<td>1.5M</td>
<td>25,665</td>
</tr>
<tr>
<td>0.5</td>
<td>2.5M</td>
<td>62,517</td>
</tr>
<tr>
<td>0.7</td>
<td>1.5M</td>
<td>51,746</td>
</tr>
<tr>
<td>1.0</td>
<td>2.0M</td>
<td>63,538</td>
</tr>
</tbody>
</table>

Figure 4: When $\alpha \neq 0$, voting based on asset holdings or how a negative shock or compression might affect the network is often most helpful.
We design an experiment to analyze the effects of compression as a strategic decision on both the compression itself and systemic risk. For each α value, 10,000 networks are randomly generated as described in Section 3.3. Nodes on a cycle are required to cast their compression vote according the Nash equilibrium at the game's α setting. When the equilibrium is mixed, the heuristic strategy is randomly chosen by a weighted draw from among the strategies in the support. When there are multiple equilibria, the results are an average of test results on each equilibrium.

To understand the impact of a strategic decision on compression, we track several metrics: number of cycles in each network, number of cycles that are compressed, number of nodes that become insolvent, total equity. When a compression is not unanimously agreed upon, we track the length of the cycle and the number of nodes that voted positively. Results on the impact of the strategic decision on compression can be seen in Table 4.

Schuldenzucker and Seuken [2020] note that in practice, banks presented with a compression proposal generally agree to perform compression. The reason for this may be that banks are following the YES policy due to regulation or a desire to simplify balance sheets. However, we find that when the nodes in our network use local network information to make the compression decision, the majority of possible compressions are not performed. This may be because nodes put a higher premium on keeping the cycle to absorb possible losses than compressing to limit exposure to insolvent nodes.

Compression is performed only when the cycle nodes vote unanimously in favor; one dissenting vote vetoes the proposal. Therefore, we questioned whether the number of performed compression remained low because the majority consensus was against compression or because compression was perceived as harmful by only a small subset of the voting notes. From our results, we can see that even though on average the majority of voting nodes agree to compression, it is often not a strong majority and in several cases the majority on average do not want compression.

5.1 Systemic Risk
One measurement of systemic risk we use is the price of anarchy. To calculate the price of anarchy, we compare for a given cycle the outcome with a strategic compression decision to the outcome under the optimal decision for the network. We measure outcomes in total equity of the network, defined as:

$$ E = \sum_{i} \left[ e_i + \sum_{j} l_{ij} - L_i \right]. $$

Figure 5: Using asset information or information about how a negative shock or compression might affect the network is often very beneficial for an agent in the network.

**Table 3:** The most useful feature to an agent making the compression decision is Assets.

<table>
<thead>
<tr>
<th>α</th>
<th>Feature with Max Gain</th>
<th>Strategy with Max Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>Assets</td>
<td>Internal</td>
</tr>
<tr>
<td>0.3</td>
<td>Assets</td>
<td>External</td>
</tr>
<tr>
<td>0.5</td>
<td>Liabilities/Dynamic</td>
<td>Neg Relative Liability</td>
</tr>
<tr>
<td>0.7</td>
<td>Assets</td>
<td>Neg External</td>
</tr>
<tr>
<td>1</td>
<td>Assets</td>
<td>Neg Internal</td>
</tr>
</tbody>
</table>

The results in Table 3 show that generally the feature Assets gives the most gains. We also find that as α increases, the strategies with the maximum gain in payoff switch from preferring compression if asset holdings are strictly greater than liabilities to preferring compression if asset holdings are smaller than liabilities. This may be because a larger α value allows insolvent nodes to recover a larger portion of their assets, which they can pass on to their creditors. Therefore, when α is smaller, nodes place a higher premium on compression’s benefit of limiting contagion. But when α is larger, nodes may tend more to view the cycle as cushioning shocks. At α = 0.5, a strategy that belongs to both the Liabilities and Dynamic features is the most useful.

We can compare the results of this feature experiment to the previous one, in which we measured the feature that provided the maximum gain when all other features are available. In the cases of α ∈ {0.1, 0.7, 1}, using Asset features provide the greatest gains in both analyses. When α = 0.3 the most beneficial feature depends on what other information nodes have available to them. Nodes with access to other local information (the previous analysis) gain the most from using Dynamic features. Starting from no local information, nodes gain the most by using the feature Assets. A similar situation holds for α = 0.5.

**Table 4:** When α ≠ 0 and the compression decision is made strategically, the percentage of cycles compressed remains low.

<table>
<thead>
<tr>
<th>α</th>
<th>% Cycles Compressed</th>
<th>% “YES” when no compression</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
<td>–</td>
</tr>
<tr>
<td>0.1</td>
<td>32</td>
<td>53</td>
</tr>
<tr>
<td>0.3</td>
<td>56</td>
<td>62</td>
</tr>
<tr>
<td>0.5</td>
<td>12</td>
<td>41</td>
</tr>
<tr>
<td>0.7</td>
<td>30</td>
<td>39</td>
</tr>
<tr>
<td>1</td>
<td>36</td>
<td>53</td>
</tr>
</tbody>
</table>
Then we can define the total equity of the optimal decision as:

\[ E^* = \max \{ E^c, E^{nc} \}, \]

where \( E^c \) is the total equity in the network where cycle \( c \) is compressed and \( E^{nc} \) is the total equity in the network where \( c \) is not compressed.

The price of anarchy is then:

\[ \text{PoA} = \frac{E^s}{E^*}, \]

where \( E^s \) is the total equity in the network where the decision to compress \( c \) comes from a strategic vote.

Figure 6 shows that the average price of anarchy is high and in fact close to 1 for all \( \alpha \) values. Thus, the strategic choice is often the optimal choice for the network and becomes closer to optimal as the recovery rate of insolvent nodes increases. When \( \alpha = 0 \) the price of anarchy is 1, since the optimal strategy is to always compress and compression in this case is never harmful.

Another indicator for systemic risk is whether the strategic compression decision was beneficial, neutral, or harmful to the network. Our first method is to classify these cases in accordance with the definitions of Veraart [2019]. Let \( SC \) be the set of nodes that become insolvent in the network with a strategic compression decision, and \( NC \) the set of nodes that become insolvent in the same network without compression. The strategic decision is beneficial and reduces systemic risk if \( SC \subseteq NC \); neutral if \( SC \subseteq NC \), and harmful if \( SC \setminus NC = \emptyset \).

A second method, following Schuldenzucker and Seuken [2020], defines social benefit in terms of total equity instead of the set of defaulting nodes. As above, let \( E^s \) the total equity in the network where compression is decided by strategic vote and \( E^{nc} \) the total equity in the same network without compression. The strategic decision is beneficial if \( E^s > E^{nc} \), neutral if \( E^s = E^{nc} \), and harmful if \( E^s < E^{nc} \).

The results of categorizing each network’s resulting compression by effects on systemic risk are in Figure 7. When using the resulting set of insolvent nodes as the metric for effects on systemic risk, most of the strategic decisions had a neutral effect. In these cases, either compression did not affect the set of banks becoming insolvent or the strategic vote resulted in no compression. When the strategic vote resulted in compression with an effect on the network, in very few cases was this effect negative. However, when looking at the total equity remaining in the system after resolving the network as the metric, the number of cases where the strategic compression is beneficial or harmful to the network increases. Although in most situations the same set of nodes are becoming insolvent, the amount of equity in the system varies.

We can further examine the difference in number of networks where strategic compression was considered beneficial by total equity to those considered beneficial by the set of insolvent nodes. It must be the case that there are networks where the effect is beneficial by total equity, but not by the set of insolvent nodes. Though we note that not every network where the effect was beneficial by insolvent nodes is beneficial by total equity.

The results in Table 5 show that such networks are overwhelmingly classified as neutral by the set of insolvent banks. Thus, while compression is changing the amount of equity in the system, it is often not affecting the set of nodes that become insolvent. Performing some amount of compression in the network, while not saving the insolvent nodes from defaulting, is able to increase the amount of equity among the remaining nodes. This may be due to a decrease in exposure of the remaining nodes to the insolvent ones as a result of compression.
Table 5: When a network is classified as beneficial by total equity only, it is generally classified as neutral by the set of insolvent nodes.

<table>
<thead>
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<th>alpha</th>
<th>beneficial</th>
<th>neutral</th>
<th>harmful</th>
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<tr>
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<td>6167</td>
<td>0</td>
</tr>
<tr>
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<td>500</td>
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<td>0</td>
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<td>590</td>
<td>4814</td>
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<td>1544</td>
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<tr>
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<td>234</td>
<td>2629</td>
<td>27</td>
</tr>
<tr>
<td>1</td>
<td>22</td>
<td>359</td>
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</tr>
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</table>

6 CONCLUSION

Our study examines the strategic decision facing nodes in a financial network presented with opportunities to remove cycles of debt by a process of debt compression. We focus on the implications for financial contagion, anticipating how compression may insulate or expose an institution to the effects of an asset shock. Taking a perspective ex ante to the potential shock, we ask what types of strategies nodes with imperfect information should use to make such a decision. Our model captures uncertainty regarding the shock and the financial situation of other nodes in the network. We compare strategies that use different types of local information that may be available to an individual node.

From our results, we can confirm that adopting a heuristic strategy based on privately held information is often better than unconditionally agreeing or disagreeing for an individual node. In particular, features related to the asset holdings of a node are found to be most useful. When these heuristics are employed, compression proposals are often declined. While based on a stylized model, our results suggest that accepting compression proposals based solely on simplifying balance sheets should be questioned, as institutions may have sufficient basis to account for risks as well as benefits of compression.

The study also illustrates a novel use of EGTA methodology to characterize the most important features for strategic decision making. In our case, this revealed an interesting pattern as we varied specific scenarios. Our analysis in this study treats the compression decision as a one-shot game. An interesting direction for future work would be to extend this model in time, considering a sequence of compression opportunities, asset shock events, debt network evolution, and so on. By extending the model in this way, we could capture signals in the network may be able to gather over time about the stability of the other nodes in the network, similar to a bank’s ability to use past payment information to make borrowing and lending decisions.

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REFERENCES


